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On the interaction of particles and turbulent fluid flow

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Abstract—A mathematical model for turbulent two-phase flows is proposed to take into account the effects of both mean and turbulent motion of each phase on the other. The modeled conservation equations are based on a Eulerian approach for the gas and a stochastic Lagrangian approach for the particles. These equations are solved numerically to predict a turbulent round gaseous jet laden with solid particles. Results demonstrate that the model is successful in predicting the significant effects of particles on both mean and turbulence quantities of the carrier phase, and the stochastic approach yields reasonably good predictions of the effects of the gas turbulence on particle dispersion.

1. INTRODUCTION

TO IMPROVE the accuracy of gas turbine combustion models, further developments are needed for modeling spray injection, dynamics, and evaporation processes. In the region where droplets are already formed, proper interaction between the carrier phase and the dispersed phase must be considered in the model formulation. It is the purpose of this paper to present such a model and to test it against reported data for particle-laden jet flow. The model accounts for the effects of turbulent fluctuations on interface quantities, turbulent dispersion of particles due to gas velocity fluctuation, and gas turbulence modulation caused by the particles.

There are basically two approaches that have been pursued for predicting particle-laden flows. For flows with particle mass loading, defined as the mass flow rate of the particles compared with that of air at the nozzle exit, less than 0.1, the gas flow field characteristics can be assumed to be unaffected by the presence of particles. In this case, the governing equations of the carrier phase have no extra terms, but rather they are identical to the Navier–Stokes equations. This approach is referred to as one-way coupling [1, 2] from gas to particles only and has been used by many workers [3–5]. When the mass loading ratio is high, the coupling between the gas and particles becomes two way [2, 6] where the particles act as sources of mass, momentum, and energy for the gas and the gas controls the motion of particles. In two-way coupling treatments, the dispersed phase calculations can be performed by employing a Lagrangian or Eulerian approach. In the Lagrangian approach, the dispersed phase is treated by solving Lagrangian equations of motion for a group of particles with a prescribed set of initial conditions. Once the flow properties of the particles are known, the interface quantities between the two phases can be calculated. The Eulerian approach treats the dispersed phase as an interacting and interpenetrating continuum which makes the governing equations of the two phases very similar to the Navier–Stokes equations, with additional source/sink terms. In the present work, a two-way coupling treatment, based on the Lagrangian approach for the particles, is used.

It is well known that even relatively small amounts of dispersed particles cause a significant change in the turbulence structure of the carrier phase. A few investigators have considered this effect by invoking many phenomenological approximations, with an attendant large number of empirical constants that render their schemes inapplicable to general flow conditions and configurations [7, 8]. Melville and Bray [9] and Michaelides [10] employed the mixing length hypothesis to handle the gas–solid flow in free jets and fully-developed pipe flows. Their approach is limited to flows where turbulence structure changes at a slow rate in the main flow direction. Two different sets of empirical constants were required to achieve agreement with measurements in free jets and fully-developed pipe flow, indicating a severely restrictive application of this approach to complex flows. Danon *et al.* [11] used a one-equation turbulence kinetic energy (K) model to consider the effects of particles on the carrier phase turbulence quantities. To obtain accurate predictions for two-phase turbulent jet flows, they multiplied the production and the dissipation terms of K by coefficients that are dependent on particle size and concentration. Encouraging results were obtained in refs. [12–14], and recently by Chen and Wood [15] by using two-equation turbulence models. These models will be compared with our model in the following sections.

The majority of the two-phase flow models mentioned are based on Eulerian approaches for the dispersed phase. This approach has serious limitations,

NOMENCLATURE

c_μ, $c_{\varepsilon 1}$, $c_{\varepsilon 2}$, $c_{\varepsilon 3}$ coefficients in the turbulence model
C_D drag coefficient
d particle diameter
D nozzle diameter
F interphase friction coefficient
g gravitational acceleration
K kinetic energy of turbulence
l_e eddy size
m particle mass
$\dot{m}$ particle mass flux
N number of particles represented by the trajectory k
P static pressure
r distance in the radial direction
Re Reynolds number
t_i, t_o times when the particle enters and leaves the carrier phase control volume
Δt time the particle takes to cross the control volume
U, u, $\tilde{U}$ mean, fluctuating and instantaneous velocity of the carrier phase
V, $\tilde{V}$ mean and instantaneous velocity of the particles
ΔV control volume used in the carrier phase solution
z distance in the axial direction.

Greek symbols
ε kinetic energy dissipation rate
μ dynamic viscosity of the carrier phase
ν_t kinematic eddy viscosity of the carrier phase
ρ material density
σ_k, σ_ε coefficients in the turbulence model
τ_d particle dynamic relaxation time
τ_e turbulent eddy lifetime
τ_L carrier phase Lagrangian time scale
τ_r residence time of the particle in the eddy
Φ volume fraction.

Subscripts
0 conditions at the nozzle exit
1 carrier phase
2 dispersed phase
c conditions at the jet centerline
i ith direction
r radial direction
z axial direction.

Superscript
k kth trajectory of a computational particle.

Abbreviations
DT deterministic treatment
LDA laser Doppler anemometer
LR mass flow rate of the particles compared to that of air at the nozzle exit
r.m.s. root-mean-square of the velocity fluctuation
SMD Sauter mean diameter
ST stochastic treatment.

especially for predictions of the dome region of gas turbine combustion chambers [16–18]. As a result, the Lagrangian approach has been widely used in modeling liquid sprays produced by fuel nozzles. However, most previous reported work using a Lagrangian approach neglected the effects of the suspended particles on the gas turbulence which means that the turbulence closure models of the single-phase flows have been used in the two-phase flow calculations. The results, presented later, indicate that this approximation could lead to serious errors and show the potential of our model in simulating the effect of particles on gas turbulence structure.

In the Lagrangian approach, the particle dispersion caused by gas turbulence must be incorporated through an empirical diffusion velocity or more realistic Monte Carlo method [19]. This latter method requires simulation of the instantaneous gas flow field to solve for the particle trajectory. Gosman and Ioannides [20] and Solomon *et al.* [21] split the turbulent gas field into mean (U_i) and fluctuating velocity components (u_i). During each droplet's flight U_i is randomly sampled and allowed to influence its motion. The cloud properties, including number density and mean velocity and temperature, are obtained by averaging over a statistically significant sample of particles. Shuen *et al.* [22] showed that the Monte Carlo method for predicting the dispersed phase provides good comparisons with their data base. In the present study, the Monte Carlo, or stochastic method, is used for modeling the effect of gas turbulence on particle motion.

The objective of this paper is to present a mathematically simple model for dilute turbulent two-phase flows. This model considers both mean and fluctuating interactions between the two phases. Both the considered data and model predictions indicate that the gas turbulence–particles interaction is equally important to the corresponding mean momentum interaction. In the following sections, the modeled conservation equations for both phases are presented using a Eulerian approach for the gas and a stochastic

Lagrangian treatment for the particles. These equations account for the effects of both mean and turbulent motion of each phase on the other. The proposed model is then tested by comparing the predictions with available experimental data of a turbulent isothermal gaseous jet laden with solid particles. Conclusions and final remarks are provided in the last section.

2. MATHEMATICAL MODEL

It is assumed that the particles are sufficiently dispersed so that particle–particle interaction is negligible. This assumption restricts the present study to dilute particulate suspensions. It is also assumed that the mean flow is steady and the material properties of the two phases are constant.

The equations of motion of the particles are cast in the Lagrangian form, while the carrier phase transport equations are formulated following the Eulerian treatment. The governing equations of the two phases are coupled primarily by the momentum interchange and the extra energy dissipation due to the relative velocity fluctuation between the gas and particles.

2.1. *Particle equations*

The discrete particles approach is considered in this study. As such, the dispersed phase is represented by computational particles rather than a continuous distribution function. This amounts to a statistical (Monte Carlo) formulation of the problem since a finite number of particles is used to represent a very large number of particles present in the field. Each of these computational particles characterizes a group of physical particles possessing the same characteristics, such as size, velocity and temperature.

It is more convenient to write the instantaneous equation of motion for each particle, rather than the averaged equation. This equation is coupled with the carrier phase through its instantaneous velocity. For a large particle–gas density ratio, the only important forces on a particle are the inertia, drag, and gravity, in which case the equation of motion of the kth computational particle in the ith direction is [16]

$$\frac{\mathrm{d}V_i^k}{\mathrm{d}t} = \frac{(\tilde{U}_i - \tilde{V}_i^k)}{\tau_\mathrm{d}} + g_i \tag{1}$$

where

$$\tau_\mathrm{d} = \frac{4d^k\rho_2}{3C_\mathrm{D}\rho_1|\tilde{U} - \tilde{V}^k|} \tag{2}$$

and

$$\tilde{U}_i = U_i + u_i.$$

The particles are assumed spherical so that the experimental results for the drag coefficient of a solid sphere can be used. For a moderate Reynolds number; $1 < Re^k < 260$, there are a lot of experimental results for the drag coefficient, and the plot of these data vs Reynolds number is called the standard drag curve. The best approximation for that curve is expressed by the relationships [23]

$$C_\mathrm{D}^k = (24/Re^k)(1 + 0.1315[Re^k]^{0.82-0.05w}),\quad 0.01 < Re^k \leqslant 20 \tag{3}$$

$$C_\mathrm{D}^k = (24/Re^k)(1 + 0.1935[Re^k]^{0.6305}),\quad 20 < Re^k \leqslant 260 \tag{4}$$

where $w = \log_{10} Re^k$ and the particle Reynolds number is calculated from

$$Re^k = \rho_1|\tilde{U} - \tilde{V}^k|d^k/\mu_1 \tag{5}$$

where

$$|\tilde{U}| = \sqrt{(\Sigma\tilde{U}_i^2)} \tag{6}$$

$$|\tilde{V}^k| = \sqrt{(\Sigma(\tilde{V}_i^k)^2)}. \tag{7}$$

The particle location at any instant of time is determined from

$$\frac{\mathrm{d}x_i^k}{\mathrm{d}t} = \tilde{V}_i^k. \tag{8}$$

In equation (1), U_i is obtained from the solution of the mean flow equations of the carrier phase. Consistent with the use of the K–ε model for the carrier phase, u_i is chosen randomly from an assumed isotropic Gaussian distribution with mean square deviation $2/3K$. Subsequently, after each elapsed time equal to turbulent characteristic time τ, a new value for u_i is chosen. τ is the minimum of turbulent eddy lifetime (τ_e) and the residence time of the particle in the eddy (τ_r) [24]. It is assumed that the characteristic length of the turbulent eddies is that of the dissipation length scale, l_e, given by [20]

$$l_\mathrm{e} = c_\mu^{3/4}K^{3/2}/\varepsilon \tag{9}$$

where ε represents the turbulent kinetic energy dissipation rate and c_μ is a constant of value 0.09. The eddy lifetime is obtained from

$$\tau_\mathrm{e} = l_\mathrm{e}/|u_i|. \tag{10}$$

The residence time of the particle in the eddy, e.g. the time for a particle to pass through that eddy, is estimated from

$$\tau_\mathrm{r} = l_\mathrm{e}/|\tilde{U} - \tilde{V}^k|. \tag{11}$$

Hence

$$\tau = \min(\tau_\mathrm{e}, \tau_\mathrm{r}). \tag{12}$$

For each particle, the equation of motion is integrated over as many time increments as required for the particle to traverse the required distance. When a sufficiently large number of particles is tracked, their averaged behavior should represent the cloud and yield the effects of the gas turbulence characteristics on the motion of particles.

2.2. *Momentum transfer between the phases*

The source term of particles in the carrier phase mean momentum equations is due to the drag force and it is composed of two parts: the change of momentum of the particles and the influence of all external forces acting on the particles. Some authors using the Lagrangian approach, such as Boyson and Swithenbank [4], Crowe [6] and Shuen *et al.* [22], define the particle source term as the net efflux of particle momentum that can be obtained by integrating the left-hand side of equation (1) over the residence time of the particle in a control volume. Durst *et al.* [25] indicated that this treatment can lead to an erroneous solution of the equations of the carrier phase because the interaction of the particles and the gas is realized through the drag force and not only through the change of particle momentum. When particles reach their settling velocities, their momentum remains constant but the exchange of momentum with the surrounding fluid, owing to the force of gravity, still exists. Another example is a fully-developed two-phase pipe flow, the source terms of particles are due to the drag force and not to the momentum efflux of particles. Although the consideration of the left-hand side of equation (1) simplifies the evaluation of the particle source terms in the gas mean momentum equations, it complicates the calculation of the corresponding terms in the equations of the turbulence kinetic energy and its dissipation rate. Recently Shuen *et al.* [22], following such a procedure, obtained a turbulence model which did not show the effects of adding solid particles on the carrier phase turbulence quantities.

In the present work, and in contrast to refs. [4, 6, 20, 22], the right-hand side of equation (1) is considered for calculating the source terms in mean momentum, turbulence kinetic energy, and its dissipation rate equations.

For each particle passing through an arbitrary control volume (ΔV) the momentum source term in the carrier phase equations can be obtained as follows:

$$S_i^k = \frac{1}{\Delta t}\int_{t_i}^{t_o} m^k \frac{(\tilde{U}_i - \tilde{V}_i^k)}{\tau_d}\,dt. \tag{13}$$

If average particles properties are considered over Δt, equation (13) can be rewritten as

$$S_i^k = m^k(U_i - V_i^k)/\tau_d. \tag{14}$$

The carrier phase momentum source per unit volume resulting from all particle trajectories is obtained as follows:

$$S_i = \sum_k \frac{N m^k (U_i - V_i^k)}{\Delta V \tau_d} = \sum_k F^k \Phi^k (U_i - V_i^k) \tag{15}$$

where the volume fraction of the particles is given by

$$\Phi^k = N\frac{\pi (d^k)^3}{6\Delta V} \tag{16}$$

and the interphase friction coefficient is given by

$$F^k = \frac{3\rho_1}{4d^k} C_D^k |U - V^k|. \tag{17}$$

2.3. *Carrier phase equations*

The governing equations of the carrier phase to be presented here are based on the model of ref. [16], with a few more assumptions suitable for mathematical manipulation. That model was devised for isothermal, non-reacting, dilute gas–droplet turbulent shear flows and was presented in its general form using Cartesian tensor notations. A preliminary version of that model was reported in ref. [17] to predict turbulent gaseous jet flows laden with evaporating droplets and should be consulted for details of the approach and model assumptions. Only those assumptions that separate the present model from the previous one [17] are presented and discussed below.

(1) In the governing equations of the carrier phase, the volume fraction of that phase (Φ_l) is approximated to unity. This is a valid assumption for dilute suspensions where the volume fraction of the dispersed phase is less than 0.01. Having used this assumption, the present turbulence model is quite simple and involves less empiricism, since all turbulent correlations between the volume fraction and gas velocity of particles are cancelled.

(2) The dispersed phase is represented by discrete particles that do not behave macroscopically as a continuum. As discussed in the previous section, this assumption allows us to use the Lagrangian approach, which is more convenient and widely used than the Eulerian approach, to solve for the particles.

(3) All third-order correlations containing particle volume fraction fluctuations are neglected. According to ref. [16], this is a valid assumption for dilute gas–particle turbulent shear flows since all the predicted cases demonstrated that the third-order correlations are at least two orders of magnitude less than the second-order ones.

(4) The mass transfer due to evapoartion is set to zero since the considered data is for jet flows laden with solid particles. Gas–particle flows are preferred for model validations over evaporating droplet–gas flows because of the large uncertainties in the measurements of the latter class. These uncertainties are due to the limitations of the experimental techniques that are currently used in measuring spray flow properties as a function of size distribution.

(5) The turbulent correlation between the relative velocity and the carrier phase component is found from the following expression [16]:

$$\overline{u_i(u_i - v_i^k)} = 2K\left(1 - \frac{\tau_L}{\tau_L + \tau_d}\right) \tag{18}$$

where τ_L is the carrier phase Lagrangian time scale given by [16]

$$\tau_L = 0.35K/\varepsilon. \tag{19}$$

The model of ref. [17] for the above correlation is based on Chao's analysis [26] of the linearized Lagrangian equation of motion of a spherical particle in a homogeneous turbulent flow. In that analysis, all forces that could act on a suspended particle in a turbulent free shear flow, such as gas pressure gradient, Bassett, and virtual mass, are retained to obtain a general solution. In the present approach, all these terms are neglected in comparison with the drag force. If a Lagrangian spectrum function equivalent to an exponential form for the velocity autocorrelation of the fluid is used, the model of ref. [17] will be reduced to equation (18).

It is noteworthy that the correlation $\overline{u_i(u_i-v_i^k)}$ can be calculated directly from the instantaneous flow properties available from the calculations of the stochastic method. However, we did not pursue that approach due to the large number of computational particles required at each gas control volume to minimize statistical errors.

Using the above assumption, the modeled conservation equations of the carrier phase are presented here in cylindrical coordinates for axisymmetric jet flow.

The mean continuity equation is

$$\rho_1 U_{z,z}+\frac{\rho_1}{r}(r,U_r)_{,r}=0. \tag{20}$$

The mean momentum equation in the axial direction (z) is

$$\rho_1 U_z U_{z,z}+\rho_1 U_r U_{z,r} = -P_{,z}+\frac{1}{r}(\rho_1 r\nu_t U_{z,r})_{,r} - \sum_k \Phi^k F^k (U_z - V_z^k). \tag{21}$$

The mean momentum equation in the radial direction (r) is

$$\rho_1 U_z U_{r,z}+\rho_1 U_r U_{r,r} = -P_{,r}-\sum_k \Phi^k F^k (U_r - V_r^k) - \frac{2}{3}\frac{\rho_1}{r}(rK)_{,r}. \tag{22}$$

In equations (20)–(22), the comma-suffix notation indicates differentiation with respect to the spatial coordinates z and r. The kinematic eddy viscosity of the carrier phase is given by

$$\nu_t = c_\mu \frac{K^2}{\varepsilon}. \tag{23}$$

The turbulence kinetic energy equation (K) is

$$\rho_1 U_z K_{,z}+\rho_1 U_r K_{,r} = \rho_1 \nu_t U_{z,r} U_{z,r}+\frac{1}{r}\left(\rho_1 \frac{\nu_t}{\sigma_k} r K_{,r}\right)_{,r} - \rho_1 \varepsilon - \sum_k 2KF^k\Phi^k\left(1-\frac{\tau_L}{\tau_L+\tau_d}\right). \tag{24}$$

The turbulence energy dissipation rate equation (ε) is

$$\rho_1 U_z \varepsilon_{,z}+\rho_1 U_r \varepsilon_{,r} = c_{\varepsilon 1}\frac{\varepsilon}{K}[\rho_1 \nu_t U_{z,r} U_{z,r}] + \frac{1}{r}\left(\rho_1 r\frac{\nu_t}{\sigma_\varepsilon}\varepsilon_{,r}\right)_{,r} - c_{\varepsilon 2}\rho_1\frac{\varepsilon}{K} - c_{\varepsilon 3}\frac{\varepsilon}{K}\sum_k\left[2KF^k\Phi^k\left(1-\frac{\tau_L}{\tau_L+\tau_d}\right)\right]. \tag{25}$$

The model constants appearing in equations (23)–(25) are [22]: $\sigma_k = 1.0$, $\sigma_\varepsilon = 1.3$, $c_\mu = 0.09$, $c_{\varepsilon 1} = 1.44$, $c_{\varepsilon 2} = 1.87$ and $C_{\varepsilon 3} = 1.0$. $c_{\varepsilon 3}$ has been optimized to produce good agreement with the data of Shuen *et al.* [22] for one loading ratio (0.2). The optimized value of this constant (1.0) is very close to that reported in refs. [27, 28].

3. NUMERICAL SOLUTION

The carrier phase governing equations are solved numerically using the marching finite-difference solution procedure of Spalding [29]. The computational grid is similar to past work [12, 17, 29, 30]: thirty-five cross stream grid nodes and marching step sizes limited to 7% of the current radial grid width or an entrainment increase of 5%, whichever is smaller.

The ordinary differential equations governing particle motion are solved using a second-order finite-difference algorithm. The total number of computational particles was progressively increased until only a 3% difference in the particle flow properties accrued between using the final number and the next highest one. Accordingly, 3000 particles are used for the stochastic treatment while 200 particles are computed when the deterministic method is compared with the stochastic one.

4. THE FLOW CONSIDERED

Shuen *et al.* [22] measured the carrier phase properties of a two-phase turbulent round jet using a laser Doppler anemometer (LDA) and the particle velocity using both LDA and shadow-photography. The jet, laden with sand particles, discharged vertically downward from a cylindrical tube of 10.9 mm diameter into a still environment. The drag coefficient for the injected sand particles was measured by determining free-fall terminal velocities using LDA. The ratio of the drag coefficient for particles of Sauter mean diameter (SMD) = 119 μm to that of a smooth sphere is 1.25. To provide a complete data set useful for the evaluation of theoretical models, a wide range of particle diameters (d) and mass loading ratios (LR) were considered. Shuen *et al.* measured the radial profiles of the mean and root-mean-square (r.m.s.) velocity and Reynolds stress at three stations (z/D = 1, 20 and 40) for both single-phase and two-phase jet flows.

Calculations are presented in this paper for single-phase jets and laden jets with particles of SMD = 119 μm at LR = 0.2 and 0.66. These calculations were started with the measured mean and r.m.s. profiles at

$z/D = 1$, except the turbulence energy dissipation rate ε. It is well known that the initial profile of ε can be calculated in a number of ways and that it directly affects the jet growth. It was assumed that the ε-profile at $z/D = 1$ remains unchanged between the single-phase and two-phase jets. To obtain agreement with single-phase data, two different approaches were attempted for the initial profile of ε. In the first approach ε was computed from the measured shear stress and velocity gradient at $z/D = 1$. The calculated profiles at $z/D = 20$ and 40 did not agree well with the data. This is probably because errors caused by small magnitudes of both velocity gradient and shear stress occur near the jet centerline at the starting plane. However, the second approach (cf. Danon *et al.* [11]), wherein ε at $z/D = 1$ was calculated from equations (26) and (27), gave a good correlation with the single-phase data

$$\varepsilon = \frac{K^{3/2}}{L} \tag{26}$$

$$L = 0.35(Y_{0.1} - Y_{0.9}). \tag{27}$$

Here L denotes a characteristic length; $Y_{0.1}$ and $Y_{0.9}$ are the cross-stream distances where the axial velocity reached 10 and 90%, respectively, of the centerline value.

5. RESULTS AND DISCUSSION

In this section, predictions using the model presented are compared with measured distributions of the mean axial velocities of both phases, particle mass flux, and the kinetic energy of turbulence and shear stress of the carrier phase. The differences between the effects of the deterministic and stochastic treatments are elucidated, as well as the effects of the additional terms in the two-phase K–ε model formulation on mean and fluctuating gas quantities.

Figure 1 shows the measured and predicted distributions of the mean centerline velocities normalized by their corresponding values at the nozzle exit ($U_{z,0}$ and $V_{z,0}$). Due to the high inertial forces of the particles compared to that of the carrier phase ($\rho_2/\rho_1 = 2300$), the centerline velocity of the particles decays at a slower rate than that of the fluid. This also could be caused by the lower turbulent diffusion of the particles compared with that of the carrier phase. In addition, Fig. 1 shows the influence of the particle loading ratio on the centerline mean velocity distributions of the carrier phase. The increase in the mean velocity of the carrier phase compared with the corresponding value of the single-phase jet is proportional to the mass loading ratio but not in a linear fashion. This behavior is analyzed in detail in ref. [12] and can briefly be attributed to two factors. The first factor is the momentum transfer from the particles to the air, since V_z^k becomes greater than U_z after a short distance downstream from the nozzle exit plane. This momentum transfer is proportional to the particle number density, or the mass loading ratio, of the dispersed phase. The second factor is that the reduction in the turbulence kinetic energy and the increase of the dissipation rate of that energy caused by the particles lead to a reduction in the radial diffusion of the carrier phase compared with that of the single phase.

Figure 1 indicates that the assumption of zero slip velocity between the particles and the gas is physically incorrect for gas–particle flows. The mean relative velocity increases with the downstream distance due

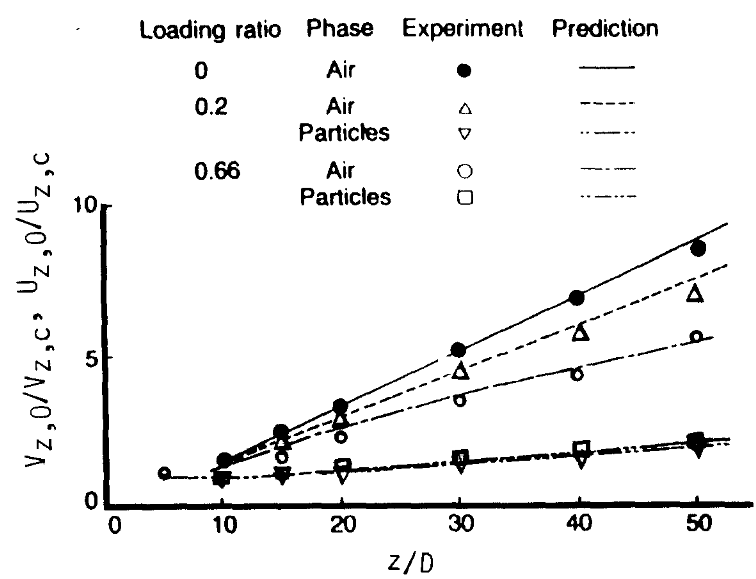

FIG. 1. Axial distribution of the mean centerline velocities.

to the high decay of the gas velocity compared with that of the particles. This unrealistic assumption has been employed in the models of Chen and Wood [15] and Danon *et al.* [11] without justification.

Figure 2 illustrates the radial profiles of the normalized mean axial velocities of the carrier phase at $z/D = 20$ and 40 for the two loading ratios LR = 0.2 and 0.66. It can be seen from this figure that the jet width decreases with the increase of the mass loading ratio. This behavior could be attributed to the same physical reasoning discussed on connection with Fig. 1.

To show how the particles modulate the turbulence structure of the carrier phase, the turbulence kinetic energy and shear stress profiles were calculated at two downstream locations—$z/D = 20$ and 40. Figure 3 shows the measured and predicted distributions of the kinetic energy of turbulence normalized by the gas mean centerline velocity. For the mass loading ratio of particles LR = 0.2, the kinetic energy is reduced by about 15%, and when the loading is increased to LR = 0.66, the reduction reaches up to 40% compared with the corresponding single-phase values. The effect of the particles on gas turbulence can also be seen in Fig. 4, where the radial profile of the turbulent shear stress is presented. The turbulence modulation shown in Figs. 3 and 4 is caused mainly by the fluctuating relative velocity between the particles and the carrier phase. This phenomenon is simulated in the present study by introducing extra terms in the turbulence kinetic energy and its dissipation rate equations.

Particles generally cause a reduction in the gas turbulence and an increase in the dissipation rate of that energy. This turbulence attenuation reaches its maximum value at a certain mass loading ratio, when the particle relaxation time becomes very large compared with the carrier phase Lagrangian time scale (see equation (18)). In fact, this condition is satisfied in most dilute gas–particle turbulent flows. In the model of Dosanjh and Humphrey [5], it was assumed that the presence of the particles has no influence on the underlying turbulent motion or, in other words, the coupling between the phases is due to the mean

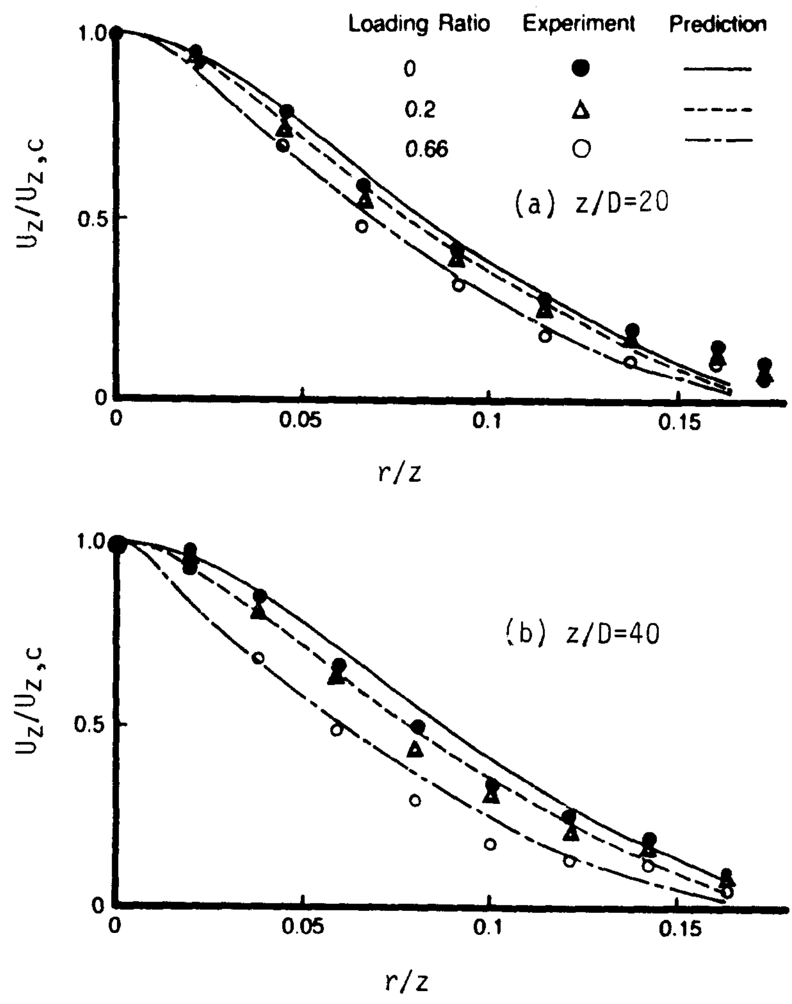

FIG. 2. Carrier phase mean axial velocity profiles.

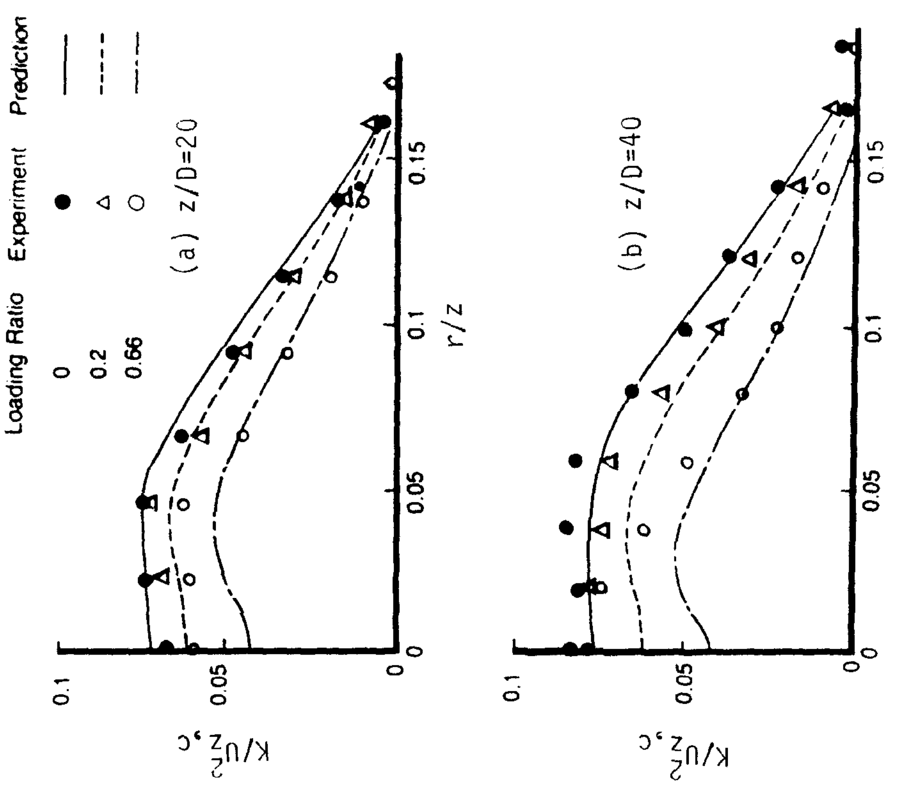

FIG. 3. Carrier phase kinetic energy profiles.

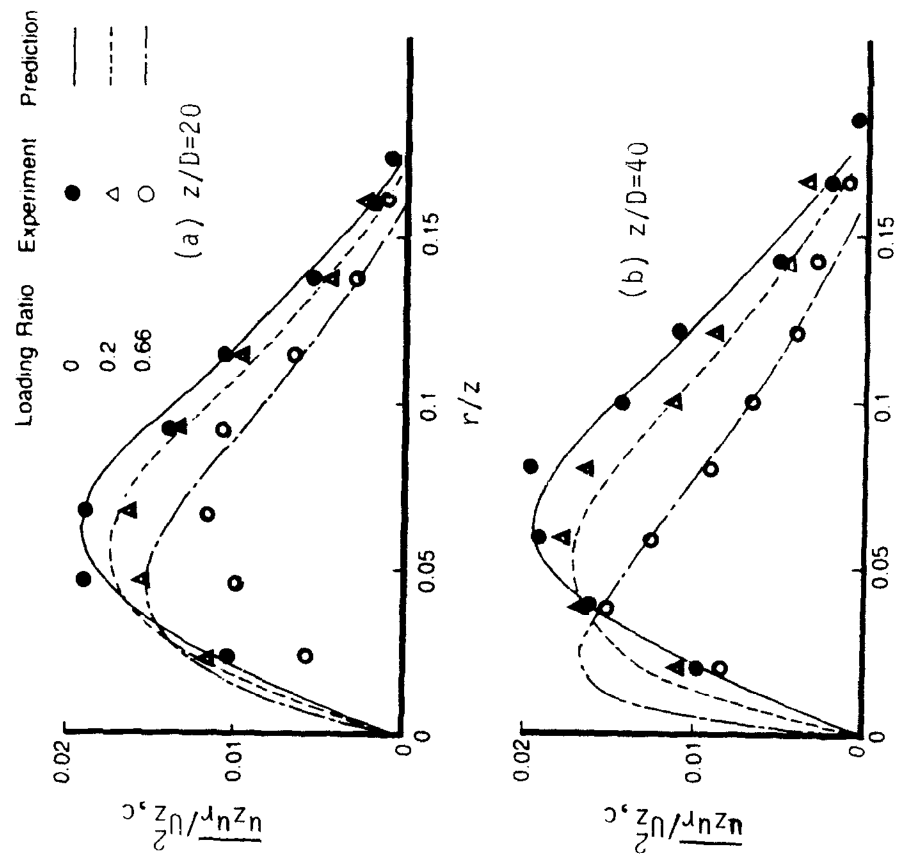

FIG. 4. Carrier phase shear stress profiles.

motion only. Figures 3, 4 and 7 suggest that an appropriate model for gas–particle flows should account for the two-way coupling caused by both relative mean and fluctuating velocities between the phases. The performance of our model, which considers this type of coupling, is very good compared with the data in Figs. 3 and 4, except for the kinetic energy of turbulence near the jet centerline, which is probably due to large discrepancies in the experimental data.

The prediction of the present model and that of Shuen *et al.* [22] for gas quantities at a loading ratio LR = 0.66 are compared with data in Fig. 5. Figure 5(a) shows the axial distribution of the mean centerline velocity while Fig. 5(b) presents the radial profiles of the mean and turbulence quantities at $z/D = 20$. Figure 5(a) indicates that the predicted mean centerline velocity of ref. [22] is underpredicted by more than 20% compared with data. This behavior is consistent with the overpredicted turbulence kinetic energy and shear stress presented in Fig. 5(b). It is noteworthy that the experimental data is consistent with the previous observations of many other workers [9, 15, 21, 27, 28]. Adding even relatively small amounts of solid particles causes a reduction in the gas turbulence kinetic energy and an increase in the dissipation rate of that energy. This turbulence attenuation reduces the jet spreading rate and causes a slower decay of the gas mean centerline velocity. In contrast to the model of ref. [22], our model yields a good prediction compared with the experimental data.

To distinguish between the effects of mean and fluctuating gas velocity on particle transport, predictions using stochastic and deterministic treatments are compared with data in Figs. 6 and 7. The first treatment considers the effect of gas velocity fluctuation (u) on the particle motion (equations (1) and (2)), while the second ignores this effect entirely. In both calculations, the turbulence model presented for two-phase flows are used. Figures 6 and 7 show the radial profiles of the normalized particle mean axial velocity and mass flux at z/D = 20 and 40 for the mass loading ratio of particles LR = 0.66. It can be seen from these two figures that the stochastic method provides good predictions compared with the experimental data, while the deterministic treatment performs quite poorly, especially for the particle mass flux. In the latter, the effects of turbulent fluctuations on particle transport are ignored and the particle only moves radially due to its initial mean radial velocity and/or the mean radial gas velocity, both of which are very small compared with the axial component. This might explain the very narrow distribution of the mass flux predicted by the deterministic treatment.

One of the interesting features of the stochastic method is the preclusion of the need to assume an effective particle diffusion coefficient necessary for the models of refs. [12, 13, 15]. Reference [16], which reviews recent developments in evaluating the turbulent diffusivity of heavy particles, showed that the experimental data on that quantity ranges over two orders of magnitude. This unsatisfactory data base has hindered the development of a reliable expression for calculating the particle diffusivity.

Although the stochastic treatment is successful in simulating the enhanced turbulent diffusion of the particle caused by gas turbulence, predictions using that treatment are not in good agreement with data, and this can be attributed to various causes. First, it is a non-trivial task to make accurate particle velocity measurements, as pointed out by Solomon *et al.* [21]. They compared double flash photographic and LDA techniques for measuring particle velocities and found that the discrepancies between the two techniques can be as high as 50% for large particles. Second, a number of assumptions that were invoked in the formulation of the present Monte Carlo method might be the reasons for the poor agreement in calculated particle velocities. These assumptions include the selection of characteristic eddy length and time scales, and uniform-property eddies in isotropic turbulence with the Gaussian distribution for velocity fluctuations. The effect of these assumptions on model predictions will be sorted out only when more data for dilute suspensions become available.

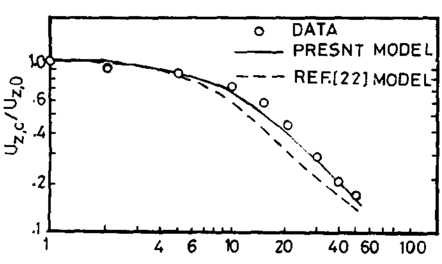

FIG. 5(a). Axial distribution of the mean carrier phase centerline velocity at loading ratio LR = 0.66.

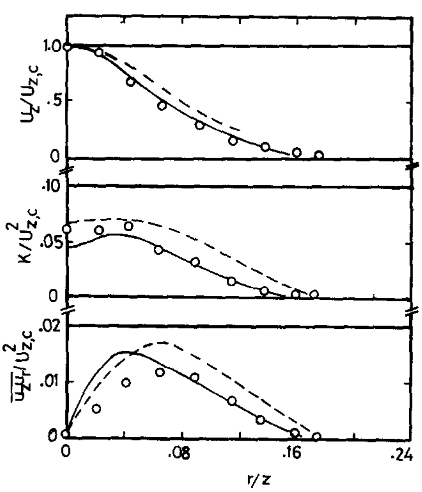

FIG. 5(b). Radial distribution of the carrier phase turbulence quantities at loading ratio LR = 0.66.

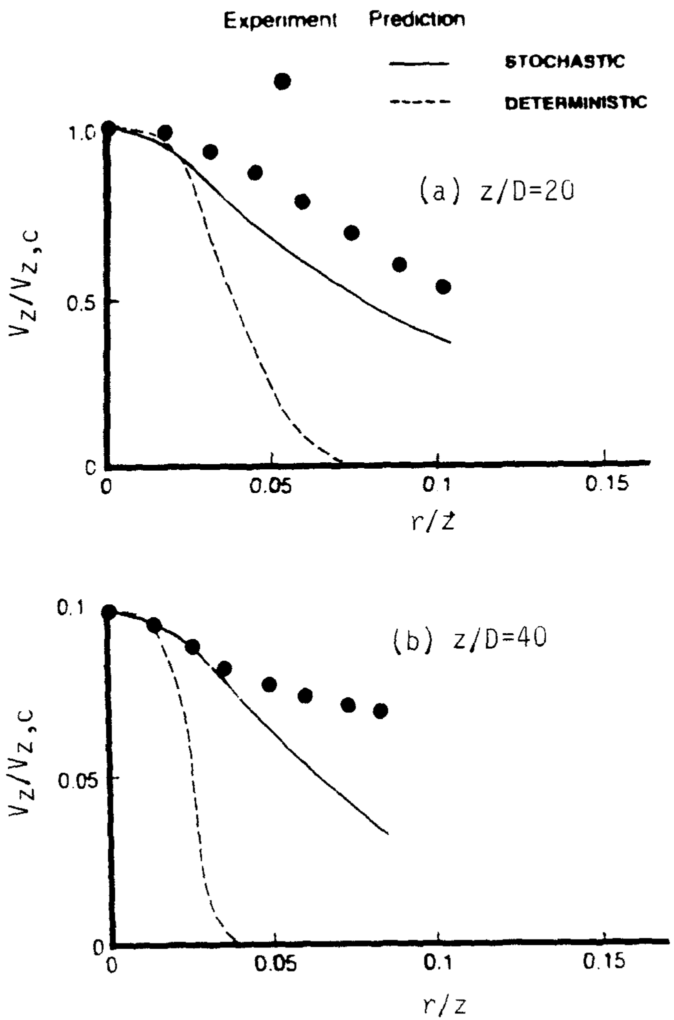

FIG. 6. Particle mean axial velocity profiles at loading ratio LR = 0.66.

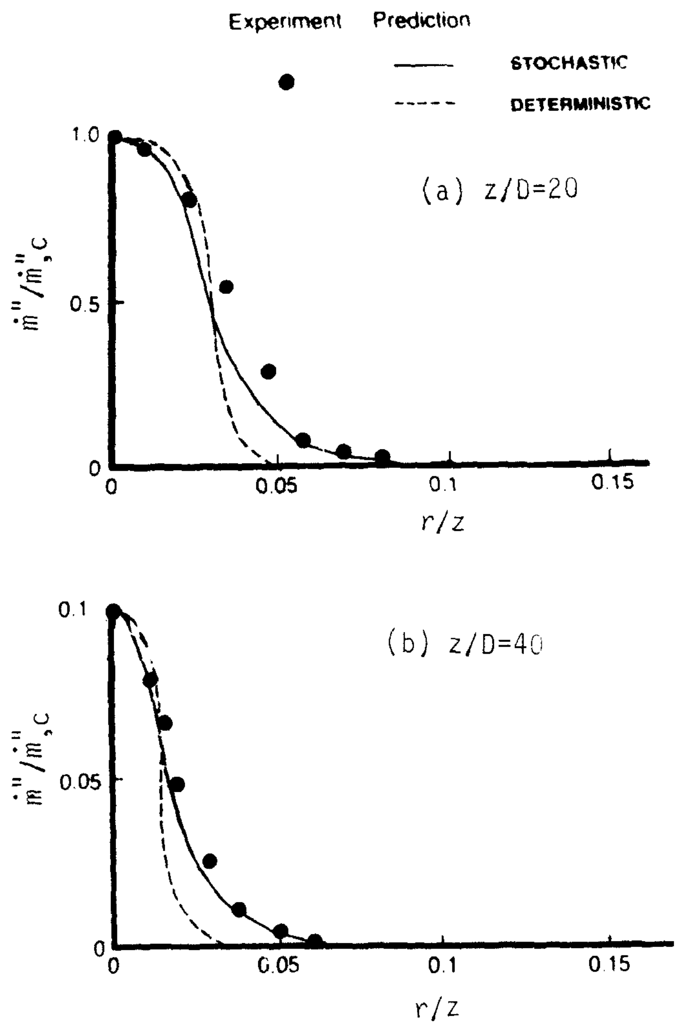

FIG. 7. Particle mass flux profiles at loading ratio LR = 0.66.

Finally, to show the effects of the additional terms in the two-phase $K\varepsilon$ model (K–ε/two-phase), predictions of the gas quantities are made using both the model presented and the one developed for single-phase flows (K–ε/standard). Figure 8 compares the performance of both models vs the experimental data of the normalized mean axial velocity and turbulence energy profiles at the downstream location $z/D = 20$, and loading ratio LR = 0.66. This figure shows that only half of the effect of particles on the gas can be predicted by using the single-phase turbulence model. This is not a surprising behavior since that model was not developed to simulate the direct effects of the particles on gas turbulence. In spite of this

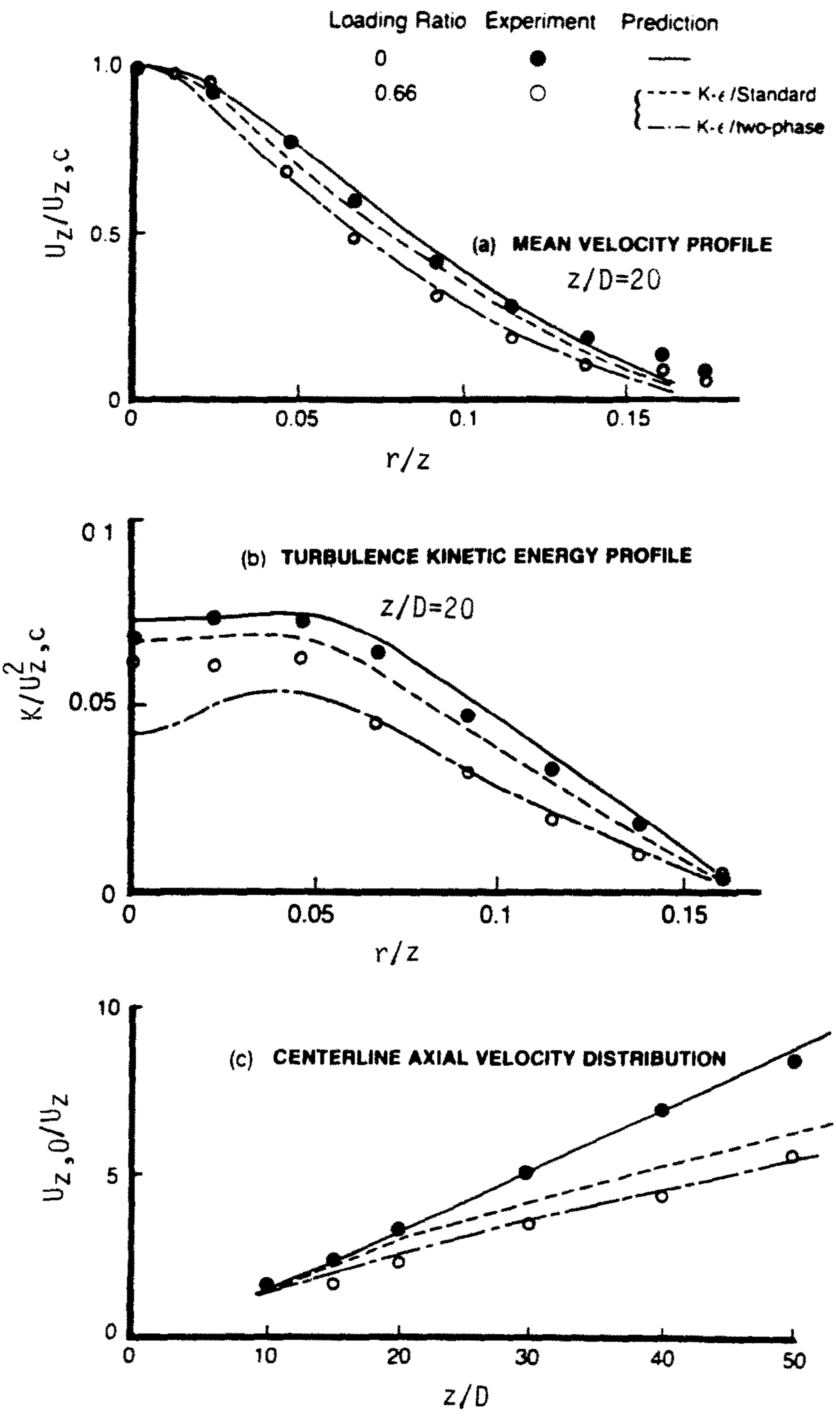

FIG. 8. Predictions of two-phase and single-phase turbulence models at loading ratio LR = 0.66.

fundamental drawback, the single-phase turbulence model has been used by Gosman and Ioannides [20] and Dosanjh and Humphrey [5] to study, respectively, the behavior of a liquid spray in a simulated gas turbine combustor and the influence of turbulence on erosion by a particle-laden fluid jet. Finally, Fig. 8, along with Fig. 1, suggests that the mean velocity of both phases cannot be assumed equal, an approximation that Chen and Wood [15] invoked in their model without justification for gas–particle flows.

Clearly the interaction between the gas and particles is due to both relative mean and fluctuating motion between the two phases. The mean relative motion is realized through the drag force while the gas turbulence that affects the particle transport senses the modulation caused by the inability of the particle to respond to the entire spectrum of the fluctuating fluid motion. This means that a heavy particle initially coincident with a fluid point naturally lags behind that fluid point and thus causes an attenuation to the turbulence eddies and enhances the viscous dissipation in its boundary region.

6. CONCLUSIONS AND FINAL REMARKS

A numerical model for dilute particle-laden turbulent flows is developed to account for the effects of suspended particles on the dissipation of carrier phase turbulence energy and the particle dispersion caused by the gas turbulence. A Eulerian approach is employed for the carrier phase subsystem of equations while a stochastic Lagrangian scheme is used for the particle equations. The model is applied to the case of a round free jet laden with solid particles. The predictions and data show significant reductions in the turbulent shear stress and kinetic energy of turbulence of the gas. This reduction is proportional to the mass loading ratio of the particles in a non-linear manner. The stochastic treatment, which allows for the effects of both mean flow and turbulent fluctuations of the gas on particle transport, yields reasonably good results over the experimental data used for comparison.

REFERENCES

1. C. Rudinger, Some effects of finite particle volume on the dynamics of gas–particle mixture, *AIAA J.* **3,** 1217–1222 (1965).
2. M. DiGiacinto, F. Sabetta and R. Piva, Two-way coupling effects in dilute gas–particle flows, ASME Paper 82-WA/FE-1 (1982).
3. G. K. Batchelor, Transport properties of two-phase materials with random structure, *Ann. Rev. Fluid Mech.* **6,** 227–255 (1974).
4. F. Boyson and J. Swithenbank, Spray evaporation in recirculating flow, Seventeenth Symposium (International) on Combustion, The Combustion Institute, Pittsburgh, Pennsylvania, pp. 443–453 (1979).
5. S. Dosanjh and J. A. C. Humphrey, The influence of turbulence on erosion by a particle-laden fluid jet, *Wear* **102,** 309–330 (1983).
6. C. T. Crowe, Review—Numerical models for dilute gas–particle flows, *ASME J. Fluids Engng* **104,** 297–303 (1982).
7. M. Nagarajan and W. Murgatroyd, A simple model of turbulent glass solids flow in a pipe, *Aerosol Sci.* **2,** 15–22 (1971).
8. Z. D. Genchev and D. S. Karpuzov, Effects of the motion of dust particles on turbulence transport equations, *J. Fluid Mech.* **101,** 833–842 (1980).
9. W. K. Melville and K. N. C. Bray, A model of the two-phase turbulent jet, *Int. J. Heat Mass Transfer* **22,** 647–656 (1979).
10. E. E. Michaelides, A model for the flow of solid particles in gases, *Int. J. Multiphase Flow* **10,** 61–77 (1984).
11. H. Danon, M. Wolfshtein and G. Hetsroni, Numerical calculations of two-phase turbulent round jet, *Int. J. Multiphase Flow* **3,** 223–234 (1977).
12. A. A. Mostafa and S. E. Elghobashi, A two-equation turbulence model for jet flows laden with vaporizing droplets, *Int. J. Multiphase Flow* **11,** 515–533 (1985).
13. F. Pourahmadi and J. A. C. Humphrey, Modeling solid–fluid turbulent flows with application to predicting erosive wear, *Int. J. Physicochem. Hydrodyn.* **4,** 191–219 (1983).
14. A. C. Buckingham and W. J. Siekhaus, Interaction of moderately dense particle concentrations in turbulent flow, AIAA Paper No. 81-0346 (1981).
15. C. P. Chen and P. E. Wood, A turbulence closure model for dilute gas–particle flows, *Can. J. Chem. Engng* **63,** 349–360 (1985).
16. A. A. Mostafa, A two-equation turbulence model for dilute vaporizing sprays, Ph.D. Thesis, University of California, Irvine (1986).
17. A. A. Mostafa and S. E. Elghobashi, A study of the motion of vaporizing droplets in a turbulent flow. In *Dynamics of Flames and Reactive Systems: Progress in Astronautics and Aeronautics* (Edited by J. R. Bowen, N. Monson, A. K. Oppenheim and R. I. Soloukhin), Vol. 95, pp. 513–539. AIAA, New York (1984).
18. H. C. Mongia and K. Smith, An empirical/analytical design methodology for gas turbine combustors, AIAA Paper No. 78-998 (1978).
19. J. K. Dukowicz, A particle–fluid numerical model for liquid sprays, *J. Computational Phys.* **35,** 229–253 (1980).
20. A. D. Gosman and E. Ioannides, Aspects of computer simulation of liquid fueled combustors, AIAA Paper No. 81-0323 (1981).
21. A. S. P. Solomon, Q.-F. Zhang, J.-S. Shuen and F. M. Faeth, A theoretical and experimental study of turbulent evaporating sprays, NASA Report No. 174760 (1984).
22. J.-S. Shuen, A. S. P. Solomon, Q.-F. Zhang and G. M. Faeth, A theoretical and experimental study of turbulent particle-laden jets, NASA CR-168293 (1983).
23. R. Clift, J. R. Grace and M. E. Weber, *Bubbles, Drops, and Particles.* Academic Press, New York (1978).
24. D. J. Brown and P. Hutchinson, The interaction of solid or liquid particles and turbulent fluid flow fields—a numerical simulation, *J. Fluids Engng* **101,** 265–269 (1979).
25. F. Durst, D. Milojevic and B. Schonung, Eulerian and Lagrangian predictions of particulate two-phase flows: a numerical study, *Appl. Math. Modelling* **8,** 101–115 (1984).
26. B. T. Chao, Turbulent transport behavior of small particles in dilute suspensions, *Osterr. Ing. Arch.* **18,** 7–21 (1962).
27. S. E. Elghobashi and T. W. Abou-Arab, A two-equation turbulence model for two-phase flows, *Physics Fluids* **25,** 931–938 (1983).
28. S. Elghobashi, T. Abou-Arab, M. Rizk and A. Mostafa, Prediction of the particle-laden jet with a two-equation

turbulence model, *Int. J. Multiphase Flow* **10,** 697–710 (1984).
29. D. B. Spalding, *GENMIX: a General Computer Program for Two-dimensional Parabolic Phenomena.* Pergamon Press, Oxford (1978).
30. A. A. Mostafa and H. C. Mongia, On the modeling of turbulent evaporating sprays: Eulerian versus Lagrangian approach, *Int. J. Heat Mass Transfer* **30,** 2583–2593 (1987).

SUR L'INTERACTION DES PARTICULES ET DE L'ECOULEMENT TURBULENT D'UN FLUIDE

Résumé—Un modèle mathématique pour les écoulements turbulents diphasiques est proposé pour prendre en compte les effets des mouvements moyen et turbulent de chaque phase sur l'autre. Les équations de conservation sont basées sur l'approche eulerienne pour le gaz et stochastique lagrangienne pour les particules solides. Des résultats montrent que le modèle est fructueux pour prédire les effets significatifs des particules de la phase vectrice, et l'approche stochastique fournit des prédictions raisonnablement bonnes pour les effets de la turbulence du gaz sur la dispersion des particules.

ÜBER DIE WECHSELWIRKUNG ZWISCHEN PARTIKELN UND TURBULENT STRÖMENDEN FLUIDEN

Zusammenfassung—Es wird ein mathematisches Modell für turbulente Zweiphasenströmungen vorgeschlagen, das sowohl die Einflüsee der mittleren, als auch der turbulenten Phasen auf die jeweils andere berücksichtigen soll. Die modellierten Erhaltungssätze basieren auf einer Euler'schen Näherung für die Gasphase und einer auf statistischen Betrachtungen beruhenden Lagrange'schen Näherung für die Partikel. Diese Gleichungen werden numerisch gelöst, um einen turbulenten, mit Partikeln beladenen, runden Gasstrahl zu berechnen. Die Ergebnisse zeigen, daß mit dem Modell die wesentlichen Auswirkungen der Partikel sowohl auf die mittleren, als auch auf die turbulenten Größen der Trägerphase erfolgreich berechnet werden können. Die statistische Näherung liefert ziemlich gute Aussagen über den Einfluß der Turbulenz des Gases auf die Ausbreitung der Partikel.

О ВЗАИМОДЕЙСТВИИ ЧАСТИЦ С ТУРБУЛЕНТНЫМ ПОТОКОМ ЖИДКОСТИ

Аннотация—Предложена математическая модель турбулентных двухфазных потоков, в которой учитывается взаимное влияние как усредненного, так и турбулентного течения каждой из фаз. Модельные уравнения сохранения основаны на эйлеровом приближении для газовой фазы и на стохастическом лагранжевом подходе для описания поведения частиц. Для турбулентной круглой струи газа с твердыми частицами модельные уравнения решены численно. Результаты показывают, что предложенная модель учитывает как существенное влияние частиц на осредненные и турбулентные характеристики несущей фазы, так и эффекты турбулентности несущей фазы на дисперсию частиц.